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13. ABSTRACT (*Maximum 200 words*)

A general method for solving several classes of problems in design of reflector/refractor systems has been developed.The method has been successfully applied to the problems of design of offset single, dual, and triple reflector systems. Numerical procedures have been also developed, investigated, and tested.

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Analysis and Synthesis of Reflector/Refractor Systems with Applications in Electromagnetics and Optics - Theory and Computational Algorithms

Final Technical Report

AFOSR Contract F49620-95-C-0014.
Contract Duration: 01 February 95 - 30 September 97

Submitted by

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November 30, 1997

Preface

The work presented in this report was performed at Matis, Inc., 1565 Adelia Place, Atlanta, Georgia 30329. This work was sponsored by the Air Force Office of Scientific Research during the period February 1, 95 - September 30, 97. The project Technical Monitor was Dr. Arje Nachman from the AFOSR. We are indebted to Dr. Nachman for all the encouragement and support he gave us during the work on this contract.

Contents

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1 Executive Summary

The geometric optics approximation is used in design of offset single and dual nonaxially symmetric reflector antennas when it is required to control the field amplitude and/or phase on the far-field or on the near-field output aperture. Similarly, various optics applications, especially laser optics, require design techniques capable of synthesizing shaped mirrors that create prespecified intensity patterns. This class of design problems is not limited to reflectors. The same type of inverse problems are encountered in design of various refractive electromagnetic and optical systems, for example, in design of radomes, lenses, and other devices for beam shaping. When formulated analytically, this problems lead to highly nonlinear partial differential equations of Monge-Ampere type to which existing methods of investigation and numerical solution can not be applied.

The main achievement of the work performed under this contract is the development of a very general geometric method for solving large classes of synthesis problems. These classes include many of the described above applications. The developed method is based on new mathematical ideas that lead to a rigorous and unified framework for establishing existence of solutions and calculating them numerically. The method has been successfully applied to the problems of design of offset single, dual, and triple reflector antennas and to the problem of creating reflectors with prespecified virtual sources.

The next natural step is to transfer the developed computational techniques into a fully operational commercial system of codes that can be utilized by design engineers. Parallel to that step it is desirable to continue to expand the classes of problems in electromagnetics and optics to which the developed method can be applied.

2 Major Accomplishments

2.1 Synthesis of single, dual, and three-reflector systems

The following results were obtained in this direction:

1. In [1] we investigated the problem of synthesis of a single reflector antenna converting the radiation pattern from a point source into a prespecified near-field energy pattern.
2. In [2] we developed and implemented a numerical algorithm for synthesis of reflector antenna systems as described in 1.

3. In [3] we established existence of solutions to the problem of synthesis of a single reflector antenna converting the radiation pattern from a point source into a prespecified far-field energy pattern.

4. In [4] we developed and implemented a numerical scheme for calculating solutions as described in 3.

5. In [5] we investigated regularity of solutions to the problem described in 3.

6. In [6] we nvestigated the problem of synthesis of offset three-reflector systems.

7. In [7] we investigated the problem of designing a reflector/refractor which transforms the energy flow from a given point source into an energy flow from a prescribed in advance set of virtual sources.

Our work on design of reflector systems with a collimated source attracted attention of optics researchers at the Physics Department of the University of Alabama at Birmingham. A group of researchers in this department led by Professors D. Shealy (the current chairman), S. Mirov, and their colleague Professor T. Basiev from Moscow Institute of General Physics is focused on developing various laser systems, including solid state multifrequency and superbroadband lasers. At the request of these researchers we designed several dual beam shaping systems transforming a Gaussian beam into a uniform beam with prespecified output shape and position. The specifications for these designs were developed by Prof. D. Shealy. In a joint effort, a series of tests to validate our designs was conducted. The researchers at UAB used an expensive commercial software package (Code V) to perform these tests. The results of these tests are in complete agreement with results predicted by our theory and numerics. The researchers at UAB expressed an interest in fabrication of the optical systems that we designed and we hope to continue this cooperation.

Let us briefly describe the main idea of our method. Let R be a smooth surface starshaped relative to some point O. Consider a point source of light positioned at O and emitting energy in directions reaching R. According to the reflection law of geometric optics a ray of direction m originating at O, is reflected off R in direction $\hat{y}$ given by

$$\hat{y} = m - 2\langle m, n\rangle n, \tag{1}$$

where n is the unit normal on R oriented so that $\langle m, n\rangle > 0$. If the surface R is a refractor then the refracted direction y is determined by Snell's law and can be written as

$$y = c_f m + (\sqrt{1 - c_f^2(1 - \langle m, n\rangle)} - c_f\langle m, n\rangle)n \ , \tag{2}$$

where c_f denotes the refraction index.

Reflecting properties of quadric surfaces are well known and have been used to build lenses, mirrors, telescopes, etc. For example, a paraboloid of revolution transforms any light

ray emanating from the focus into a ray parallel to the axis of revolution. An ellipsoid transforms any light ray from one focus into a light ray passing through the other focus. In Caffarelli and Oliker [8] we have shown that using optical properties of paraboloids of revolution it is possible to construct a convex reflecting surface such that for a given point source of light the reflected directions cover a unit sphere with prespecified in advance density. Similarly, Kochengin and Oliker [1] proved existence of a convex reflecting surface, built of ellipsoids, such that the light emanating from a point source and reflected off the reflector illumates a given target with a prespecified energy density. The fact that a light ray emanating from one focus is reflected into the other focus (which lies at infinity in case of a paraboloid) is very important for the construction of such reflectors because it allows to treat finite sets of points on the target as caustic points and build reflectors that are piece-wise quadric surfaces which direct to these caustic points prespecified amounts of energy. Once this property is established, a special approximation procedure is used to redistribute the energy over general measurable sets with prespecified density. This idea is general enough to be applicable always when there exists a quadric surface that reflects or refracts a beam of rays into a focal point.

The following three designs illustrate an application of our techniques to the problem of synthesis of reflecting surfaces that redirect and reshape the beam generated by a point source according to a given set of specifications. In all three cases the design requirements are to illuminate a specified far-field aperture with given intensity. Energy losses and blockage are prohibited. All figures represent computer generated pictures of synthesized reflectors.

In example 1 the intensity of the source is constant and the reflector is required to redirect the incident beam in the following manner. Choose a Cartesian coordinate system (x_1, x_2, x_3) with center at the source O and a system of polar coordinates (θ, ϕ), $\theta \in [0, \pi]$, $\phi \in [0, 2\pi)$ on the unit sphere S. These coordinate systems are connected by the equations:

$$x_1 = \cos(\phi)\sin(\theta), \quad x_2 = \sin(\phi)\sin(\theta), \quad x_3 = \cos(\theta).$$

Let $D = \{0 \leq \theta \leq \pi/3; \;\; 0 \leq \phi < 2\pi\}$ be the input aperture and the input intensity $I \equiv 3$. As the output aperture on the far-sphere we take the set $T = \{(\theta - \pi/4, \phi), \mid 11\pi/12 \leq \theta \leq \pi; \;\; 0 \leq \phi < 2\pi\}$. The designed reflector is shown on Fig. 1. The computed solution is presented in the table below. In this table, the first column represents the focal directions of the paraboloids from which this reflector is constructed. The second column contains the focal parameters d for these paraboloids. The required intensity distribution and the distribution actually created by the constructed reflector are represented in the third and the fourth columns. The reflector is constructed from 19 parabolic pieces. This design provides a "uniform" illumination of T without any blockage. The error in this case is bounded by 0.02.

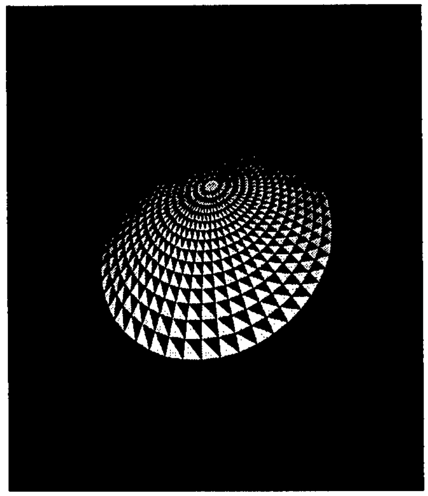

Figure 1: Design 1

axial direction	focal parameter	actual distribution	required distribution
(0.6293, 0.0000,-0.7771)	1.04531	0.5025	0.5043
(0.6662,-0.0905,-0.7401)	1.02748	0.5012	0.5043
(0.7401,-0.0905,-0.6662)	0.99104	0.5046	0.5043
(0.7771, 9.e-09,-0.6293)	0.97245	0.5011	0.5043
(0.7401, 0.0905,-0.6662)	0.99104	0.5046	0.5043
(0.6662, 0.0905,-0.7401)	1.02748	0.5012	0.5043
(0.5446, 0.0000,-0.8386)	1.11873	0.5005	0.5016
(0.5643,-0.1039,-0.8189)	1.10891	0.5014	0.5016
(0.6181,-0.1800,-0.7651)	1.08032	0.5011	0.5016
(0.6916,-0.2079,-0.6916)	1.04148	0.5019	0.5016
(0.7651,-0.1800,-0.6181)	1.00108	0.5003	0.5016
(0.8189,-0.1039,-0.5643)	0.97162	0.5012	0.5016
(0.8386, 1.e-08,-0.5446)	0.96005	0.4999	0.5016
(0.8189, 0.1039,-0.5643)	0.97162	0.5012	0.5016
(0.7651, 0.1800,-0.6181)	1.00108	0.5003	0.5016
(0.6916, 0.2079,-0.6916)	1.04148	0.5019	0.5016
(0.6181, 0.1800,-0.7651)	1.08032	0.5011	0.5016
(0.5643, 0.1039,-0.8189)	1.10891	0.5014	0.5016
(0.7071, 0.0000,-0.7071)	1.00000	0.3965	0.3790

In our second example we took as input the Gaussian intensity $I = 10e^{-3\theta}$, $T = \{(\theta - \pi/2, \phi),\ |\ 11\pi/12 \le \theta \le \pi;\ \ 0 \le \phi < 2\pi\}$, and D is the same as in example 1. The designed reflector consists of 19 paraboloids and converts the Gaussian input intensity into a uniform illumination of T. The reflector is shown on Figure 2. The computed solution is given in the table below. This reflector was found in 117 iterations with error tolerance

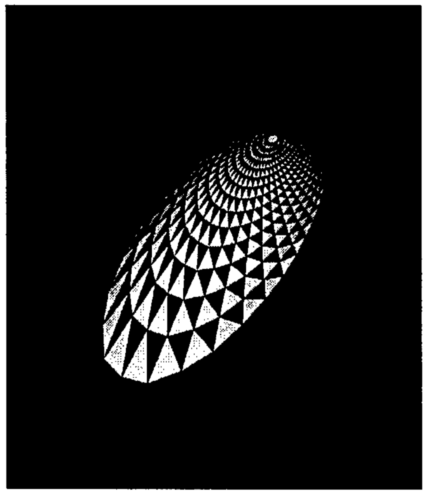

Figure 2: Design 2

bounded by 0.05.

axial direction	focal parameter	actual distribution	required distribution
(0.9945, 0.0000,-0.1045)	1.10336	0.2884	0.2955
(0.9945,-0.0905,-0.0522)	1.05647	0.2924	0.2955
(0.9945,-0.0905, 0.0522)	0.96267	0.2940	0.2955
(0.9945, 9.e-09, 0.1045)	0.91543	0.2908	0.2955
(0.9945, 0.0905, 0.0522)	0.96267	0.2940	0.2955
(0.9945, 0.0905,-0.0522)	1.05647	0.2924	0.2955
(0.9781, 0.0000,-0.2079)	1.23248	0.2948	0.2939
(0.9781,-0.1039,-0.1800)	1.20831	0.2876	0.2939
(0.9781,-0.1800,-0.1039)	1.14001	0.2939	0.2939
(0.9781,-0.2079, 5.e-08)	1.04636	0.2936	0.2939
(0.9781,-0.1800, 0.1039)	0.94967	0.2925	0.2939
(0.9781,-0.1039, 0.1800)	0.87780	0.2913	0.2939
(0.9781, 1.e-08, 0.2079)	0.85070	0.2860	0.2939
(0.9781, 0.1039, 0.1800)	0.87780	0.2913	0.2939
(0.9781, 0.1800, 0.1039)	0.94967	0.2925	0.2939
(0.9781, 0.2079, 4.e-08)	1.04636	0.2936	0.2939
(0.9781, 0.1800,-0.1039)	1.14001	0.2939	0.2939
(0.9781, 0.1039,-0.1800)	1.20831	0.2876	0.2939
(1.0000, 0.0000, 4.e-08)	1.00000	0.2711	0.2221

A modification of the last design was obtained by taking $T = \{(\theta - \pi/4, \phi), \mid 11\pi/12 \leq \theta \leq \pi;\ \ 0 \leq \phi < 2\pi\}$. This produced the reflector shown on Figure 3. Observe that in the last two examples the "apex" of the reflector is displaced. This happens because the reflected

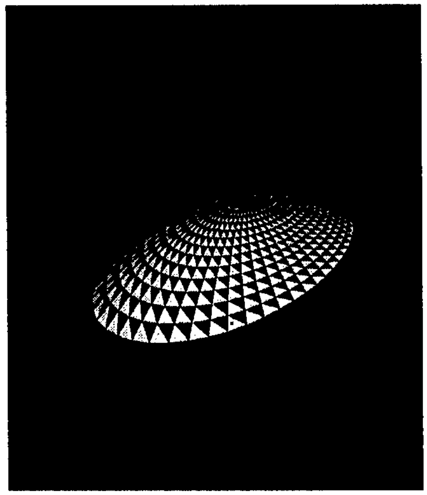

Figure 3: Design 3

beam must be directed so that its central ray forms an acute angle with the central ray of the input beam. The closer these two central rays are the closer the "apex" of the reflector is to the boundary of the reflector. For a larger angle, as an example 1, the apex "moves" towards the central ray of the input cone.

2.2 Applications of surface ray-tracing techniques to prediction of Antenna-to-Antenna and Antenna-to-Aircraft interactions

The knowledge and experience gained in our investigations of reflector systems were used to develop a system of algorithms for predicting electromagnetic interference between antennas mounted on aircraft. Specifically, Matis, Inc. developed and implemented an algorithm, Propagation Path Finder (PPF), for finding propagation paths between pairs of antennas mounted on aircrafts without any specific restrictions on the underlying geometry or location of antennas. The released version of the PPF is capable of performing required computations on a faceted file representing the actual aircraft. This R&D effort was performed for IIT Research Institute (IITRI) in Annapolis, MD, as a part of larger contract between DOD Joint Spectrum Center and IITRI.

A number of enhancements have been made to the original algorithm leading to a more efficient and reliable system. As a result of these efforts IITRI began a multi-million project of redesigning their current interference analysis system. The following quote is taken from the Statement of Work describing the FY97/FY98 effort by IITRI:

"...As a result of MATIS, Inc. success in developing the PPF algorithms and supporting

software, IITRI will start in FY97 a major software development AAPG 2000 (Aircraft inter-Antenna Propagation with Graphics). ... One of the principal components of AAPG 2000 will be the MATIS, Inc. developed Propagation Path Finder (PPF) software." Currently, Matis, Inc. continues to support the AAPG 2000 project and plays a principal role in the development.

In May of 1996 we received an SBIR Phase I Award to develop a prototype system, DOVA (Diffraction Over Virtual Airframes), for modeling and analysis of the "creeping wave" effects involving propagation over a complete airframe. The modeling and calculations are to be performed on a virtual aircraft represented by a realistic electronic model. The unique feature of this work that makes it fundamentally different from other research in this area is the capability to analyze surface wave propagation on aircraft models that are high fidelity computer representation of actual aircrafts. The usual assumptions requiring the aircraft to be modeled as a collection of plates, cones, and cylinders are not made. The aircraft models acceptable for analysis by our system are "virtual" aircrafts in the sense that they can be CAD files developed by manufacturers. This means that antenna radiation patterns, RCS's, and other EM effects predicted by systems based on our approach will be several orders of magnitude more accurate than the predictions made by many of the currently available systems.

In this particular work, a system of algorithms (**DOVA-P** - Diffraction Over Virtual Airframe - Prototype Version) for constructing geodesic surface rays on faceted surfaces representing realistic geometries has been developed and implemented into an operational computer code. The algorithms are based on geometric techniques suitable for construction of high accuracy and efficient numerical schemes. In addition, an algorithm for computing the Fock parameter, which is one of the most important electromagnetic parameters associated with the computed propagation path, has also been developed and coded. Other capabilities of DOVA-P include a user friendly interface and a 3D visualization subsystem for display of an aircraft with the computed surface ray. All computations are performed in real time; for example, determination of a typical path on a lower end Silicon Graphics computer takes less than a second.

A number of different tests validating the developed algorithms and codes have also been performed. The results of these tests are in very good agreement with results obtained by other methods.

Subsequent to the SBIR Phase I Award we received the SBIR Phase II Award. The main objective of this work is to use our techniques to develop an operational code for predicting airborne antenna radiation patterns for realistically modeled aircraft.

Most recently, we received the STTR Phase I award to develop new geometric and numerical techniques for high frequency electromagnetic propagation/scattering calculations. These techniques are based on novel approaches to ray tracing via direct geometric methods

and nonlinear partial differential equations. The new computational schemes will be used for calculation of the required geometric data, for calculation of scattering in the shadow region and elsewhere, for determination of multivalued solutions accounting for ray crossing, etc. The new techniques will be implemented into a prototype code capable of performing the calculations on fully realistic aircraft models. This work is carried out as a joint effort with a research team from UCLA.

2.3 Nonlinear Degenerate Diffusion

In addition to the above work we also continued our earlier work on diffusion flows propagating with curvature dependent speed. The following results were obtained in this direction:

1. In [9] we continued our earlier work and established that asymptotically the solutions to the degenerate parabolic problem describing mean curvature motion approaches exponentially the first eigenfunction of the Jacobi operator. This result explains the asymptotic behavior of such flows and provides an explicit estimate of the rate of convergence to stationary solutions.
2. In [10] we have shown that even without assuming the existence of a stationary solution to the mean curvature flow, the diffusion process always (under some natural assumptions) converges to a variational solution of the corresponding stationary problem. This is true, even if the resulting stationary solution has singularities and does not satisfy the boundary conditions.

These results complete a series of investigations that started with our paper [11] and continued in [12], [13], [14], [9], [10]. One of the very surprising results obtained by us here is the discovery and explanation of the fact that mean curvature flows for nonparametric surfaces may develop singularities even with analytic initial data and that such singularities disappear in finite time. This work attracted the attention of researchers working on edge detection problem in computer science. We hope to pursue this work in the future, especially, the applications in computer science.

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4 Publications

All of the above references, except for [11] - [14] resulted from the work performed under this effort.

5 Presentations

The following is a partial list of presentations on the work performed under this contract.

1. AFOSR Contractor/Grantees meeting, San Antonio, TX, January, 1995
2. Plenary lecture at the meeting on PDE's, Univ. of Koln, Germany, Febr., 1995
3. Lecture at the meeting on Qualitative Theory of PDE's, Oberwolfach, Germany, Febr.1995
4. Lecture at the meeting on CEM, ICASE/NASA, Hampton, VA, May, 1995
5. Two 1-hour Keynote Addresses at International Workshop on Computer Vision and Applied Geometry, Nordfjordeid, Norway, Aug. 1995
6. Symposium on Antenna and Propagation, U of Il., Allerton, Oct. 1995
7. Distinguished Lecturer, A series of 3 lectures delivered at the invitation of Math. Dept. of University of Toledo, Sponsored by Shoemaker Fund, April, 1996
8. Invited speaker at the International Conference on Differential Geometry, Rio-de-Janeiro, Brazil, July, 1996
9. Invited talk at the ICASE/LaRC Conference, Williamsburg, October 1996
10. Invited lecturer in the Rome Lab. Seminar Series, Dec. 1996
11. AFOSR Contractor/Grantees meeting, San Antonio, TX, January, 1996
12. Plenary Speaker, CBMS-NSF Conference on Mange-Ampere Equations, Boca Raton, Florida, July 1997
13. URSI Symposium on Antennas and Propagation, July, 1997